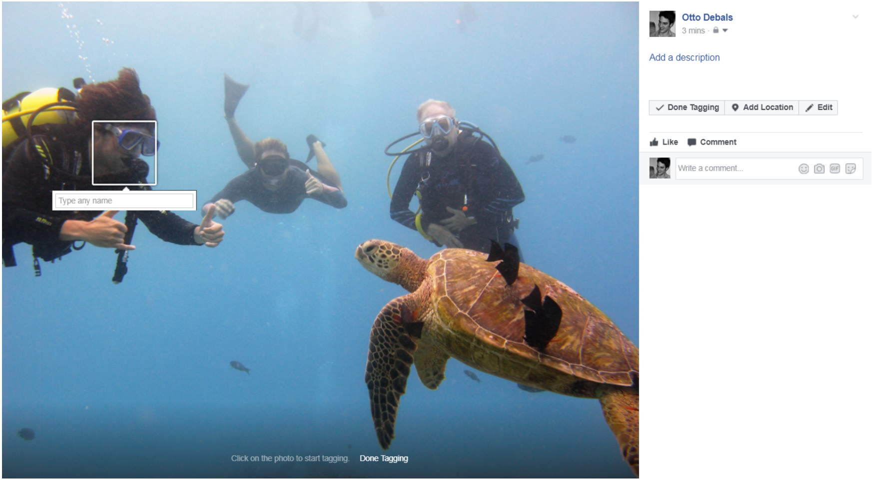
Numerical optimization algorithms for tensor-based face recognition

Otto Debals, Martijn Boussé, Nico Vervliet and Lieven De Lathauwer

How does Facebook suggest tags?



Face recognition . . .

different persons

Face recognition is a multidimensional problem

different persons

different illumination

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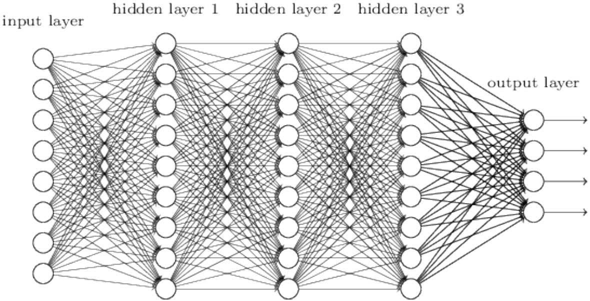
and different pose, expression, ...

Face recognition is a multidimensional problem

Linear algebra is limited to single-mode variations

Of course, nonlinear models such as neural nets can be used



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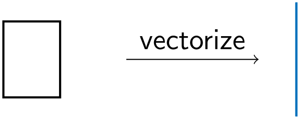

image

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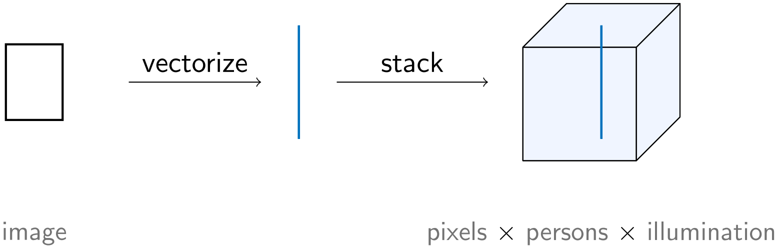

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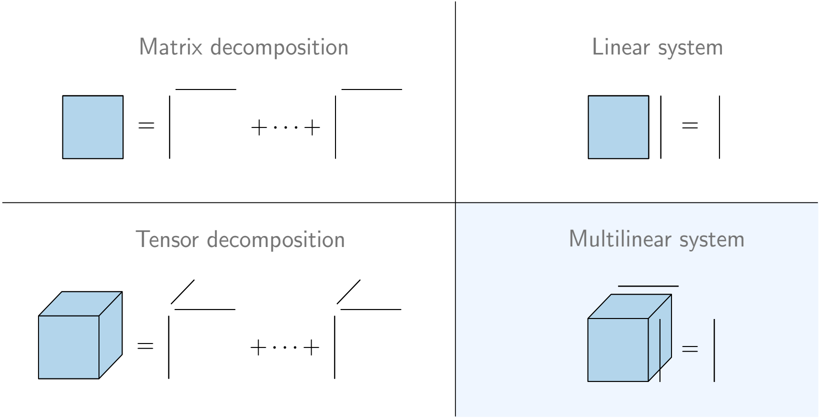


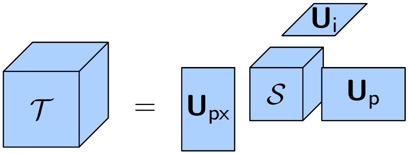
Tensor-based face recognition:

1. Construct bases using multilinear SVD (MLSVD)

Tensor-based face recognition:

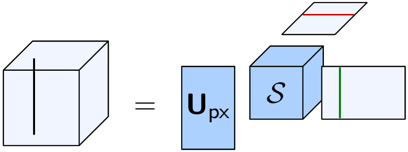
1. Construct bases using multilinear SVD (MLSVD)
2. Recognize a person using multilinear systems or Kronecker product equations (KPE)

We compute a multilinear SVD to approximately separate the influences of the different modes



$$\mathcal{T} \approx \mathcal{S} \cdot_1 \mathbf{U}_{\mathrm{px}} \cdot_2 \mathbf{U}_{\mathrm{p}} \cdot_3 \mathbf{U}_{\mathrm{i}}$$

Every (vectorized) facial image can then be expressed as follows



$$\mathbf{d} = (\mathcal{S} \cdot_1 \mathbf{U}_{px}) \cdot_2 \mathbf{c}_p^T \cdot_3 \mathbf{c}_i^T$$

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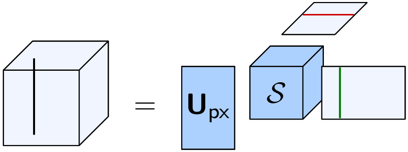

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$$\updownarrow$$

$$\mathbf{d} = (\mathbf{U}_{px}\mathbf{S}_{(1)})(\mathbf{c}_i \otimes \mathbf{c}_p)$$

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$$\mathbf{d}^{(\mathrm{new})} = (\mathbf{U}_{\mathrm{px}} \mathbf{S}_{(1)})(\mathbf{c}_{\mathrm{i}}^{(\mathrm{new})} \otimes \mathbf{c}_{\mathrm{p}}^{(\mathrm{new})})$$

and we compare the estimate of $\mathbf{c}_{\mathrm{p}}^{(\mathrm{new})}$ with the rows of $\mathbf{U}_{\mathrm{p}}$ to find the closest match and use the corresponding label.

What about multiple images of the same person?

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$$\Downarrow$$

$$\mathbf{d}^{(q)} = (\mathbf{U}_{\text{px}}\mathbf{S}_{(1)})(\mathbf{c}_{\text{i}} \otimes \mathbf{c}_{\text{p}}^{(q)})$$

The latter, which is a coupled KPE, can be written as

$$\mathbf{D} = (\mathbf{U}_{\text{px}}\mathbf{S}_{(1)})(\mathbf{c}_{\text{i}} \otimes \mathbf{C}_{\text{p}})$$

Illustration on a real-life dataset

We use the extended Yale B dataset

51×58 image
37 persons
64 illuminations

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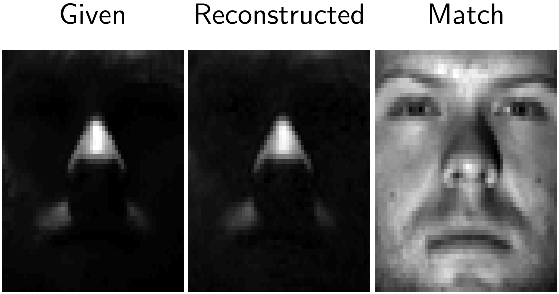
51×58 image
37 persons
64 illuminations

Of course, we use Tensorlab for all computations in MATLAB

We correctly classify a *known* person even though the image is almost completely dark



The KPE-based approach has a very competitive performance

Table: Performance (%) and recognition time (s) in comparison to conventional techniques such as 'EigenFaces' [Turk and Pentland, 1991] as well as another tensor-based approach 'TensorFaces' [Vasilescu and Terzopoulos, 2002]

	'Eigenfaces'	'Tensorfaces'	KPE
Accuracy	93.3	93.5	**95.7**
Precision	90.6	94.4	**96.6**
Recall	88.4	90.9	**95.8**
Time of PCA/MLSVD	**2.97**	3.29	3.29
Time of recognition	**0.004**	0.135	0.097

Higher performance can be achieved using multiple images under different illuminations

	'Eigenfaces'			cKPE-based method		
# illuminations	1	2	3	1	2	3
Accuracy	92.7	93.3	96.3	**95.8**	**97.1**	**97.3**
Precision	89.8	91.2	97.9	**97.0**	**99.3**	**99.9**
Recall	87.7	87.8	97.5	**96.2**	**99.2**	**99.9**

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- To 'register' a person $\mathbf{d}^{(!)}$ not in the database we solve

$$\mathbf{d}^{(!)} = (\mathbf{U}_{\mathsf{px}}\mathbf{S}_{(1)})(\mathbf{c}_{\mathsf{i}}^{(!)} \otimes \mathbf{c}_{\mathsf{p}}^{(!)})$$

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- New image $\mathbf{d}^{(?)}$ under different illumination, we solve

$$\mathbf{d}^{(?)} = (\mathbf{U}_{\mathsf{px}}\mathbf{S}_{(1)})(\mathbf{c}_{\mathsf{i}}^{(?)} \otimes \mathbf{c}_{\mathsf{p}}^{(?)})$$

  and compare $\mathbf{c}_{\mathsf{p}}^{(?)}$ to $\mathbf{U}_{\mathsf{p}}$

The MLSVD model generalizes quite well for a *new* person using only one image with a neutral illumination

Given Reconstructed

We correctly classify the *new* person using a different illumination

Given Reconstructed Best match Second match

Again, using images with multiple illuminations can improve the results

Person	One illumination	Two illuminations	Three illuminations
16	51.8	56.4	**59.3**
25	64.3	72.7	**75.9**
28	58.9	63.6	**70.4**
All	61.8	66.2	**68.1**

Kronecker Product Equations are a powerful tool for many problems, including face recognition

The generic problem of finding $\mathbf{b}$ and $\mathbf{c}$ in

$$\mathbf{d} = \mathbf{A}(\mathbf{b} \otimes \mathbf{c})$$

has many other applications [Boussé et al., 2017]:

- Classification of ECG signals
- Blind separation of telecommunication signals
- Weighted tensor decompositions
- Tensor decomposition updating
- ...

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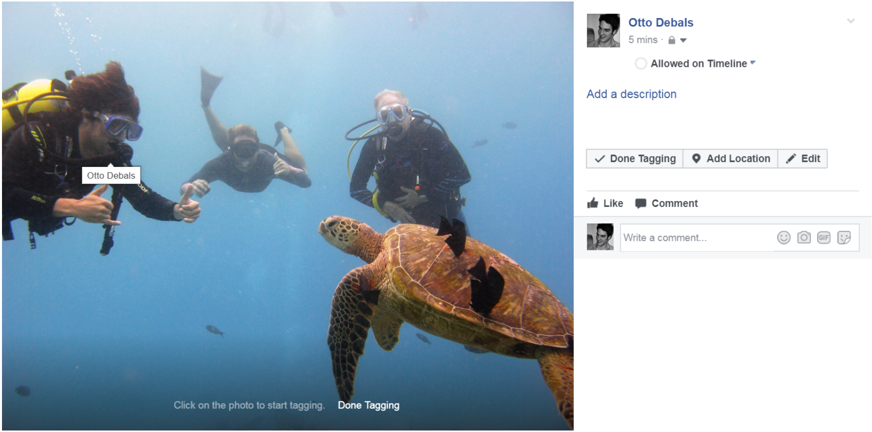

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