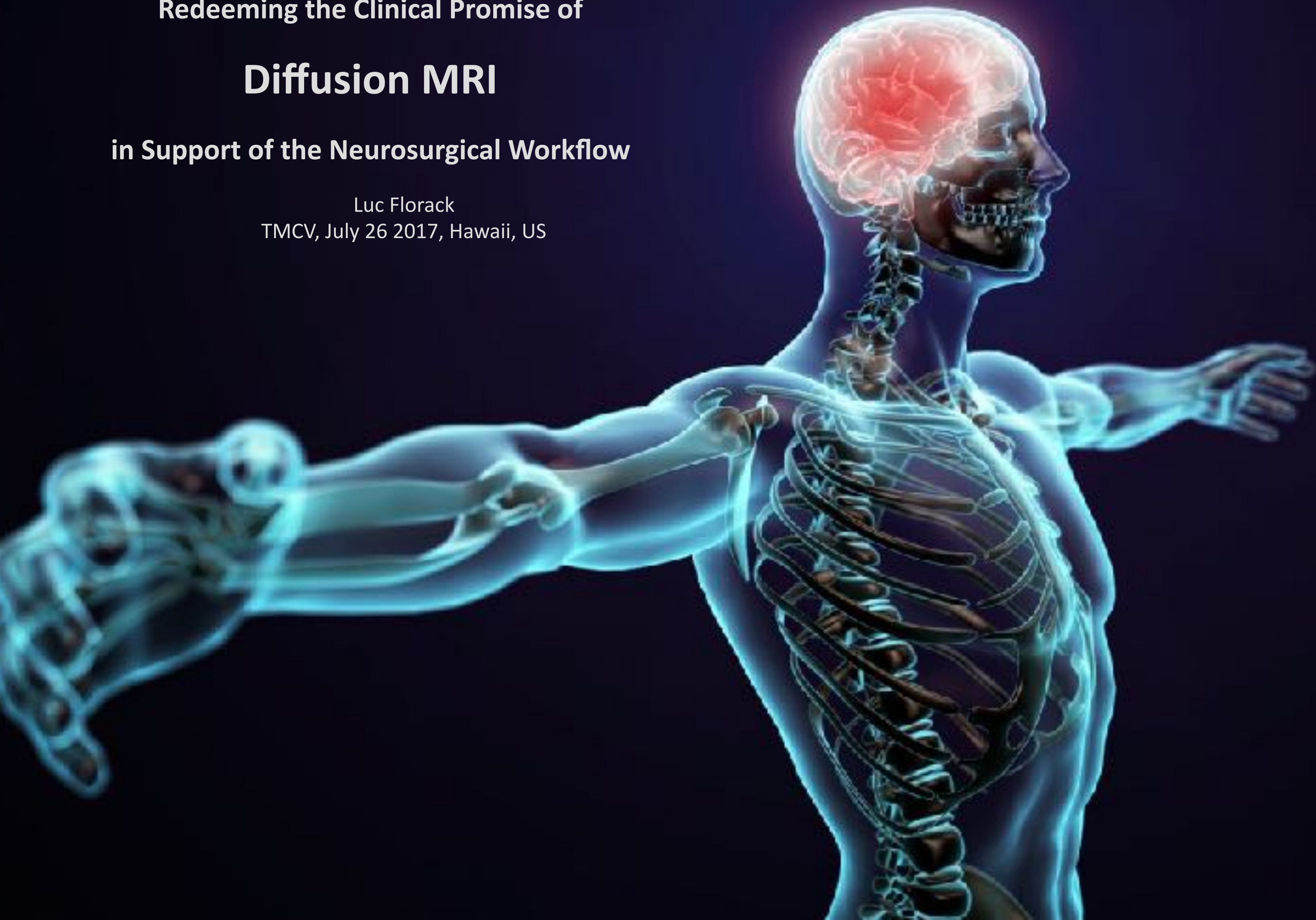
Redeeming the Clinical Promise of

Diffusion MRI

in Support of the Neurosurgical Workflow

Luc Florack

TMCV, July 26 2017, Hawaii, US



Economic Cost of Brain Disorders in Europe 2010:

€ 798 billion …

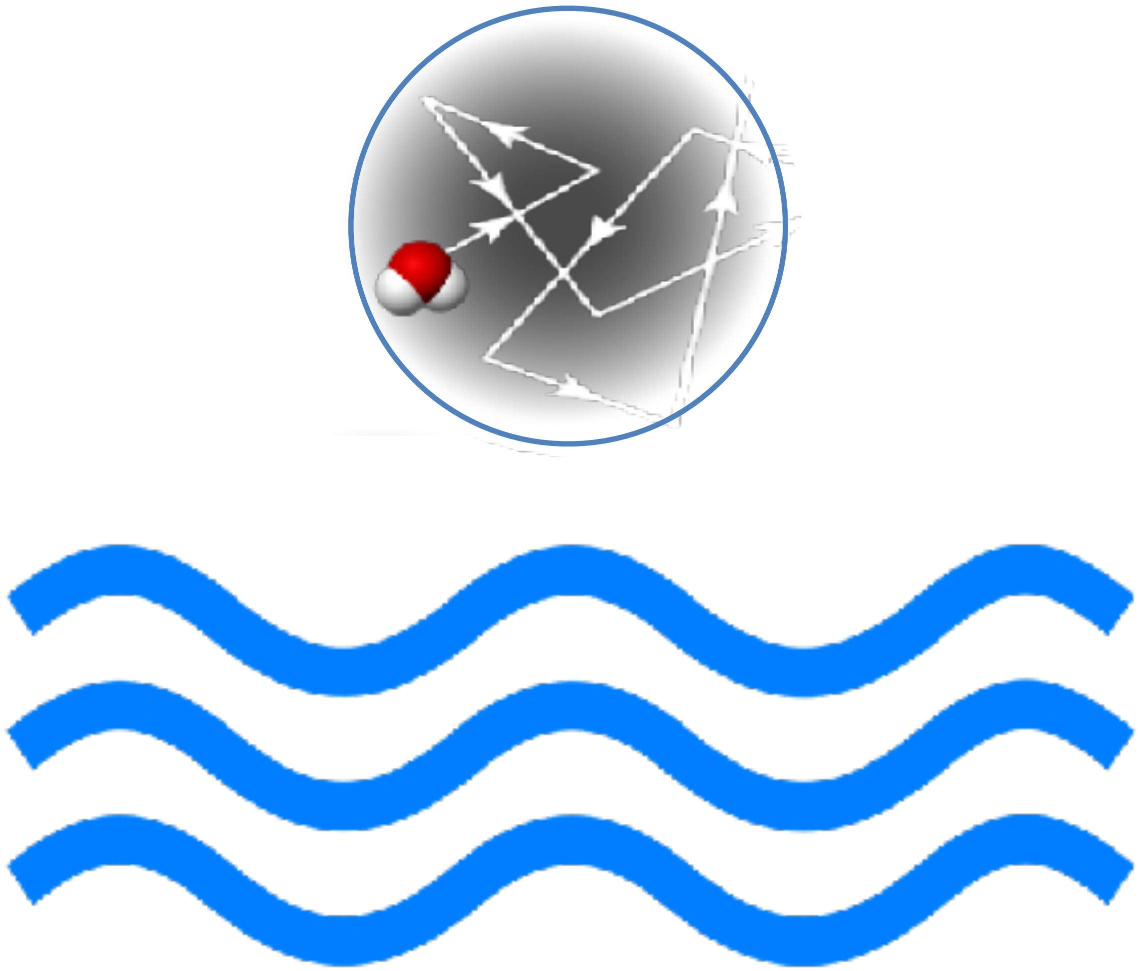
European Journal of Neurology 2012, 19: 155–162

(N)MRI scanner

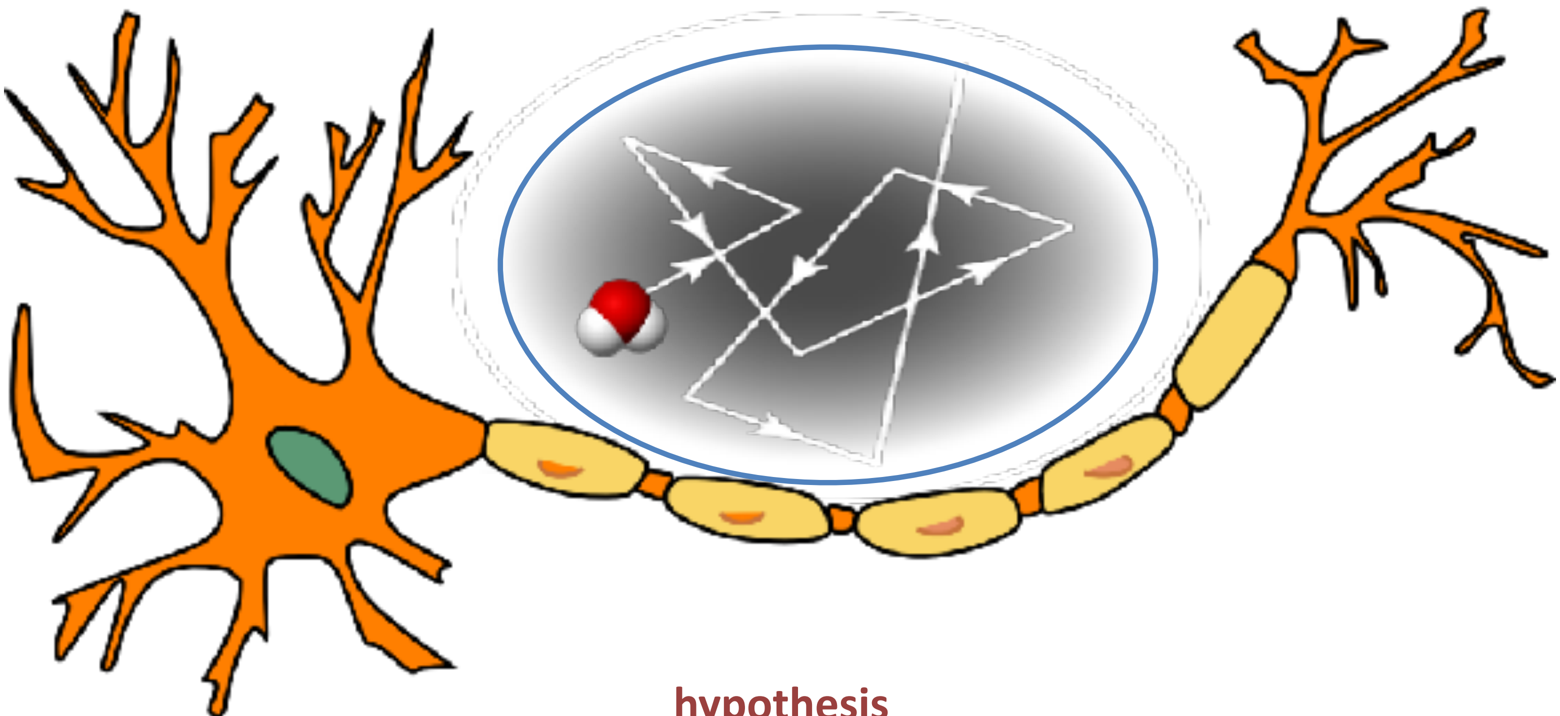
"everything must be made as simple as possible, but not one bit simpler"

attributed to Albert Einstein

operational model: brain = constrained water



operational model: brain = constrained water

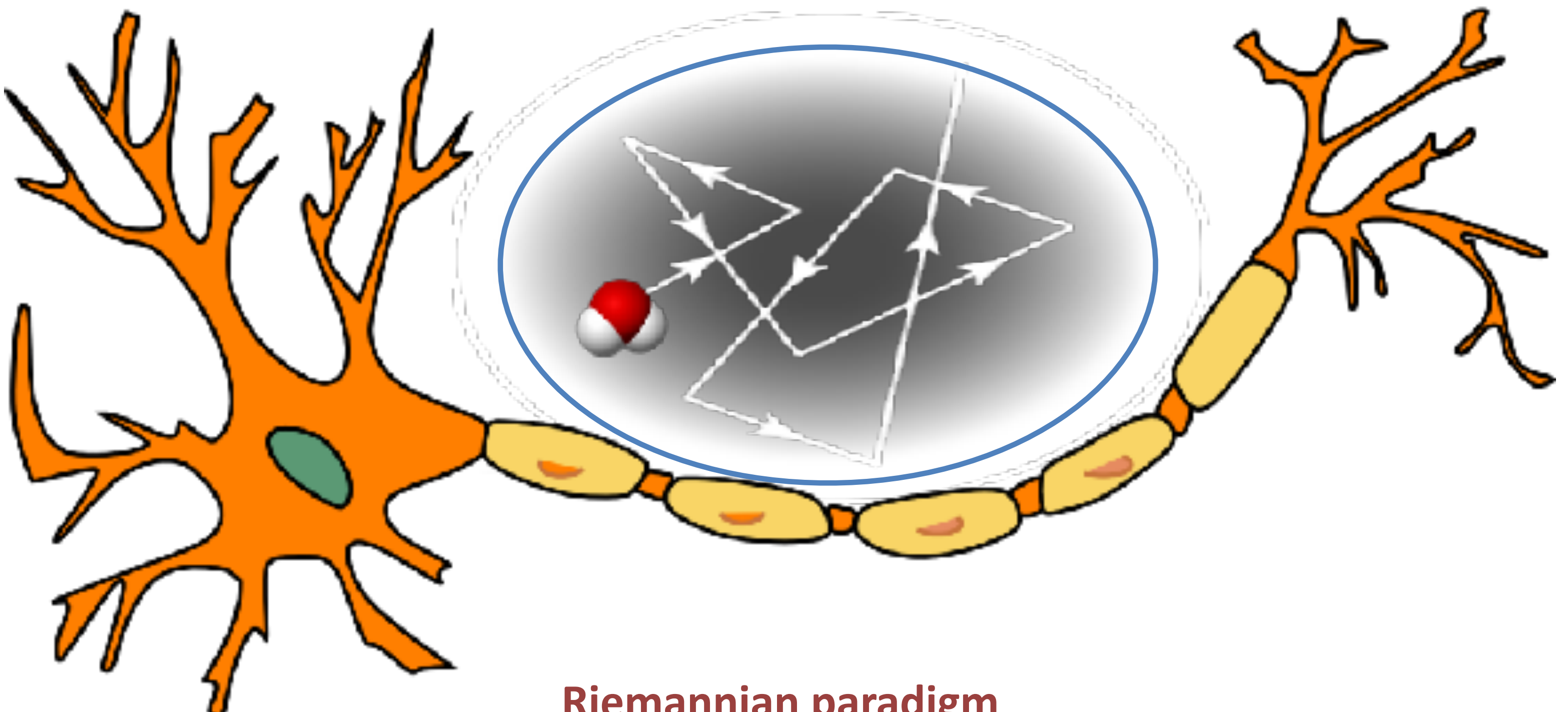


hypothesis

tissue microstructure imparts non-random barriers to water diffusion

C. Beaulieu, NMR Biomed. 2002, vol. 15, nr. 7-8, DOI: 10.1002/nbm.782

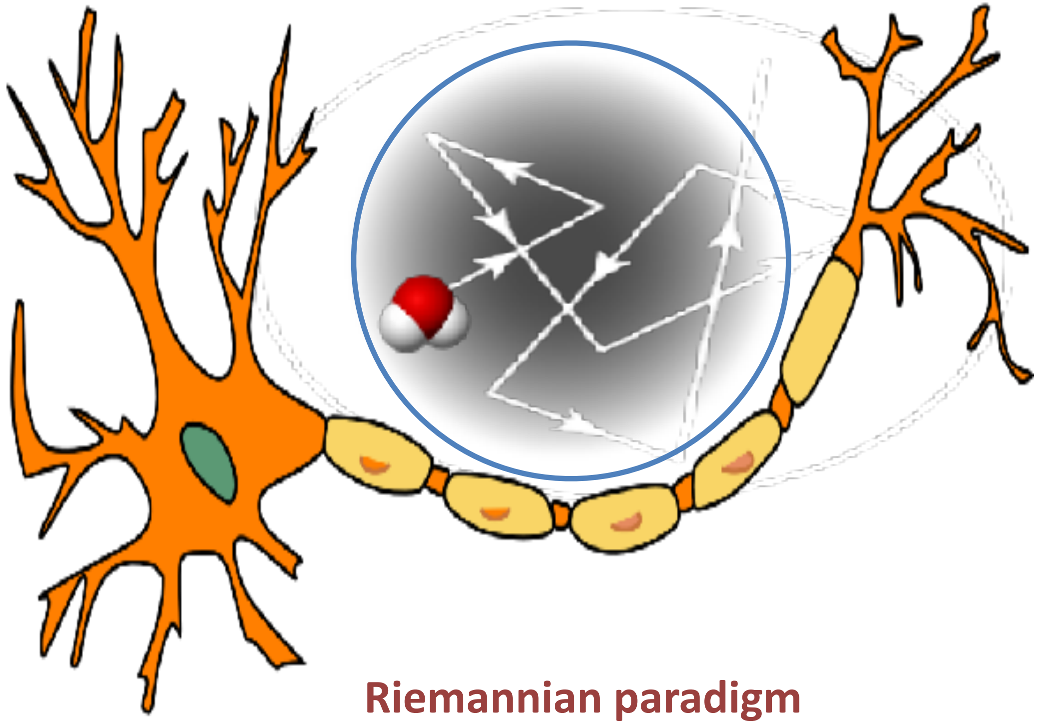
operational model: brain = constrained water



Riemannian paradigm

extrinsic diffusion on Euclidean space ≈ intrinsic geometry of a Riemannian space

operational model: brain = constrained water



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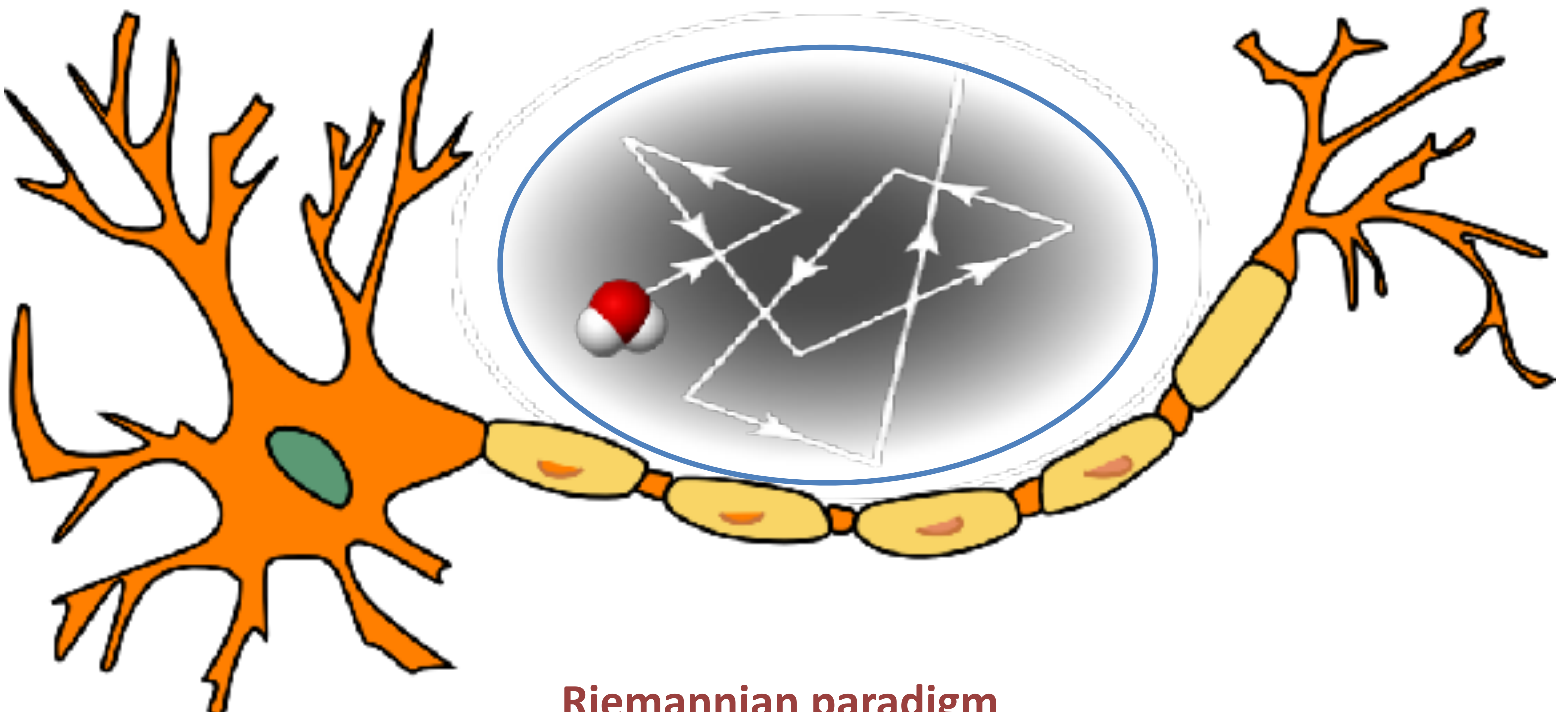

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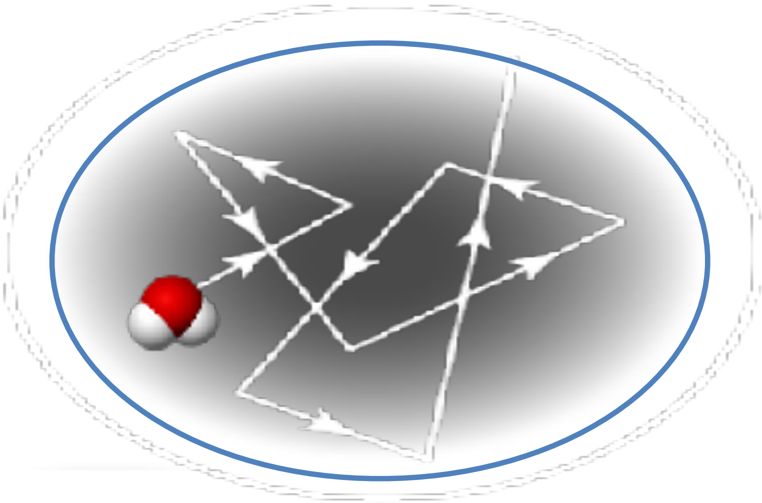
local gauge figure

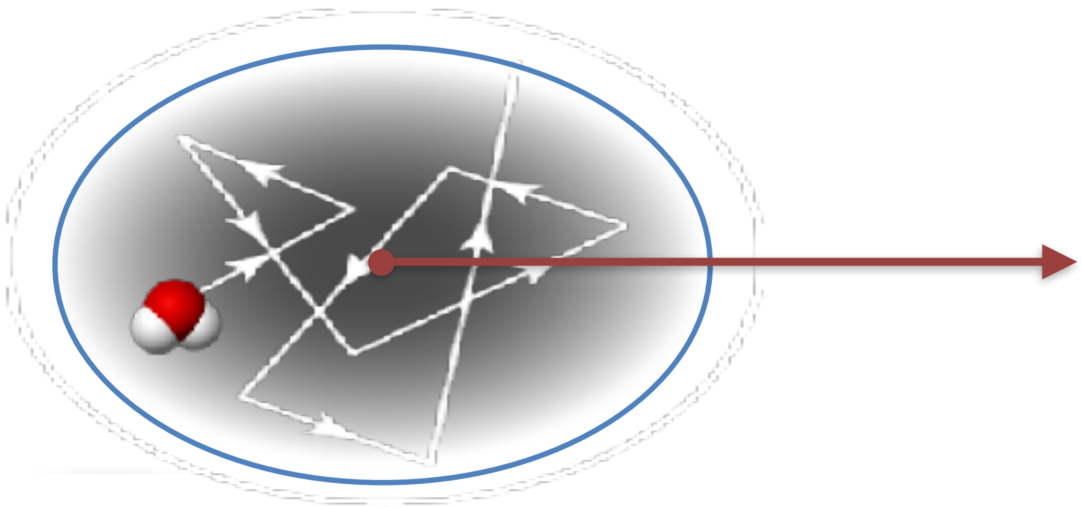
Riemann geometry



gauge figure = unit sphere = indicatrix = Riemannian metric = inner product

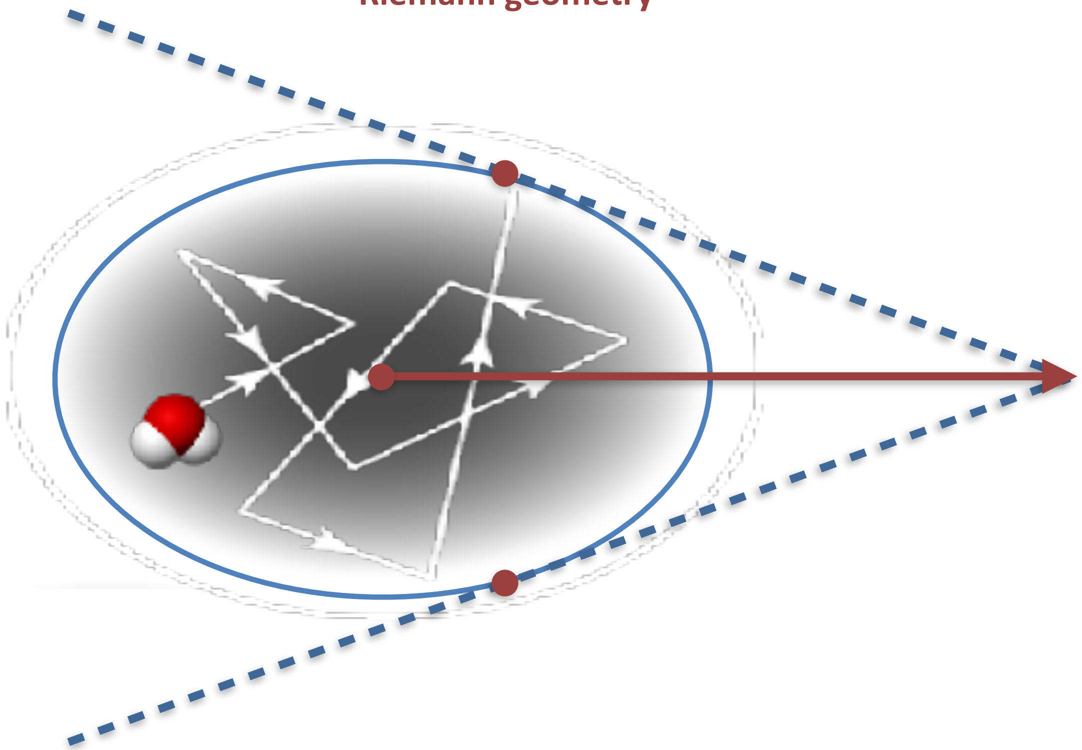
local gauge figure

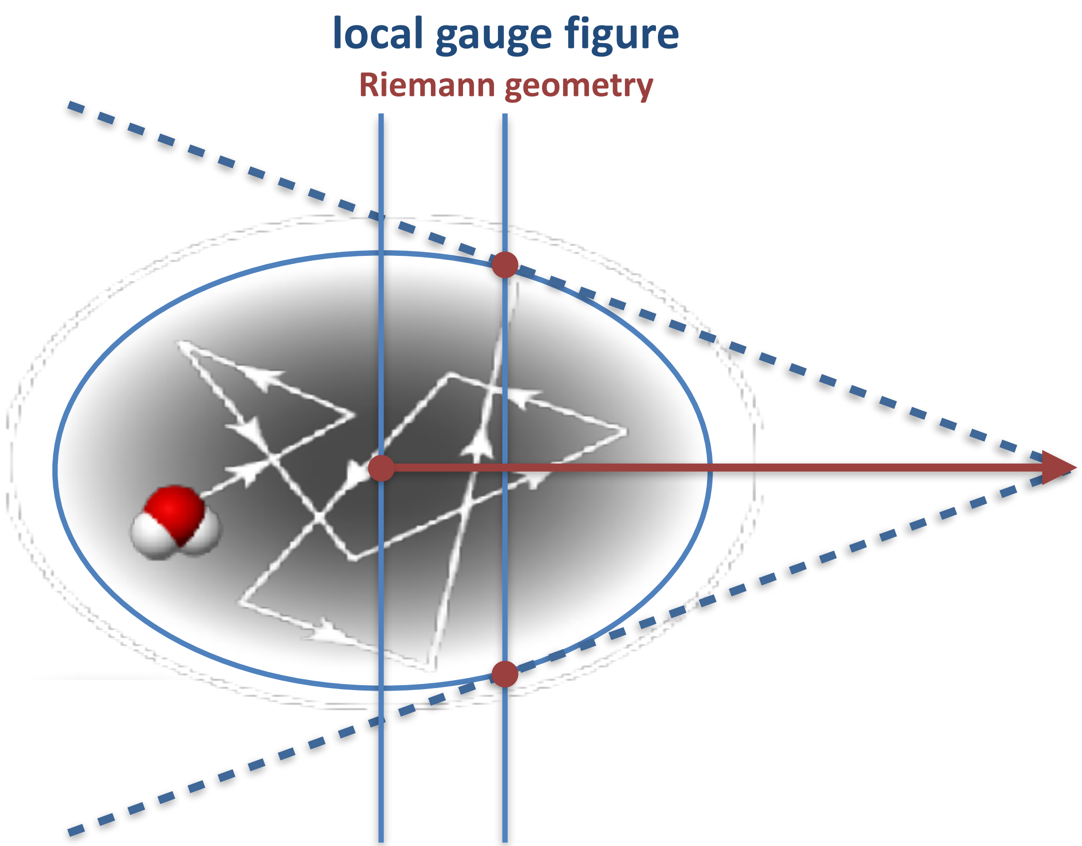
Riemann geometry



local gauge figure

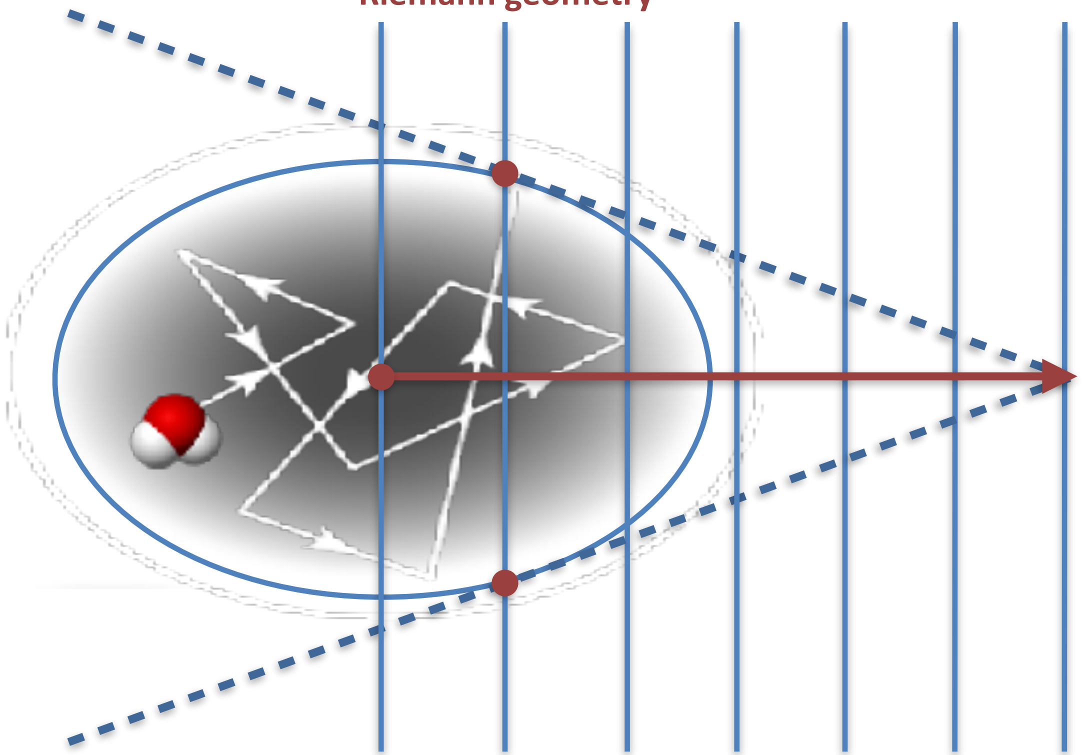
Riemann geometry




local gauge figure
Riemann geometry

local gauge figure

Riemann geometry



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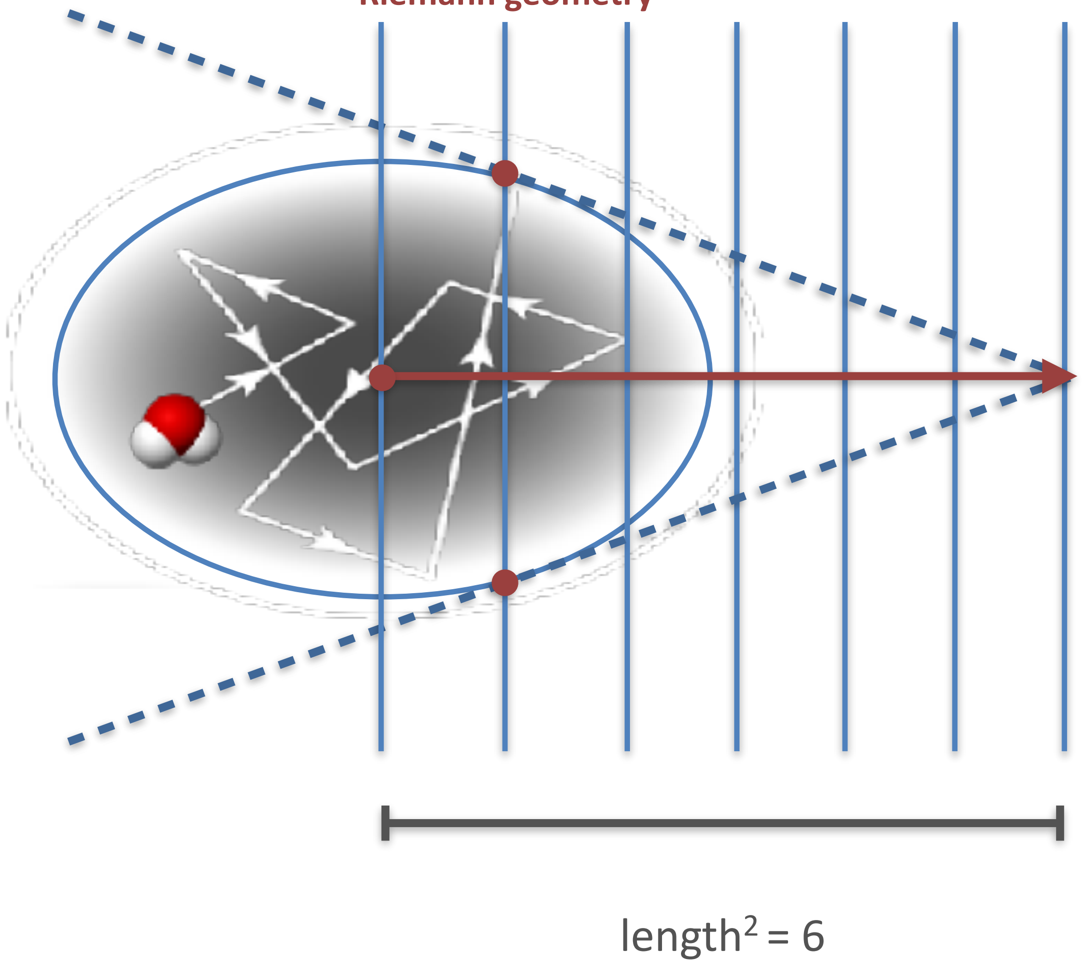
Riemann geometry



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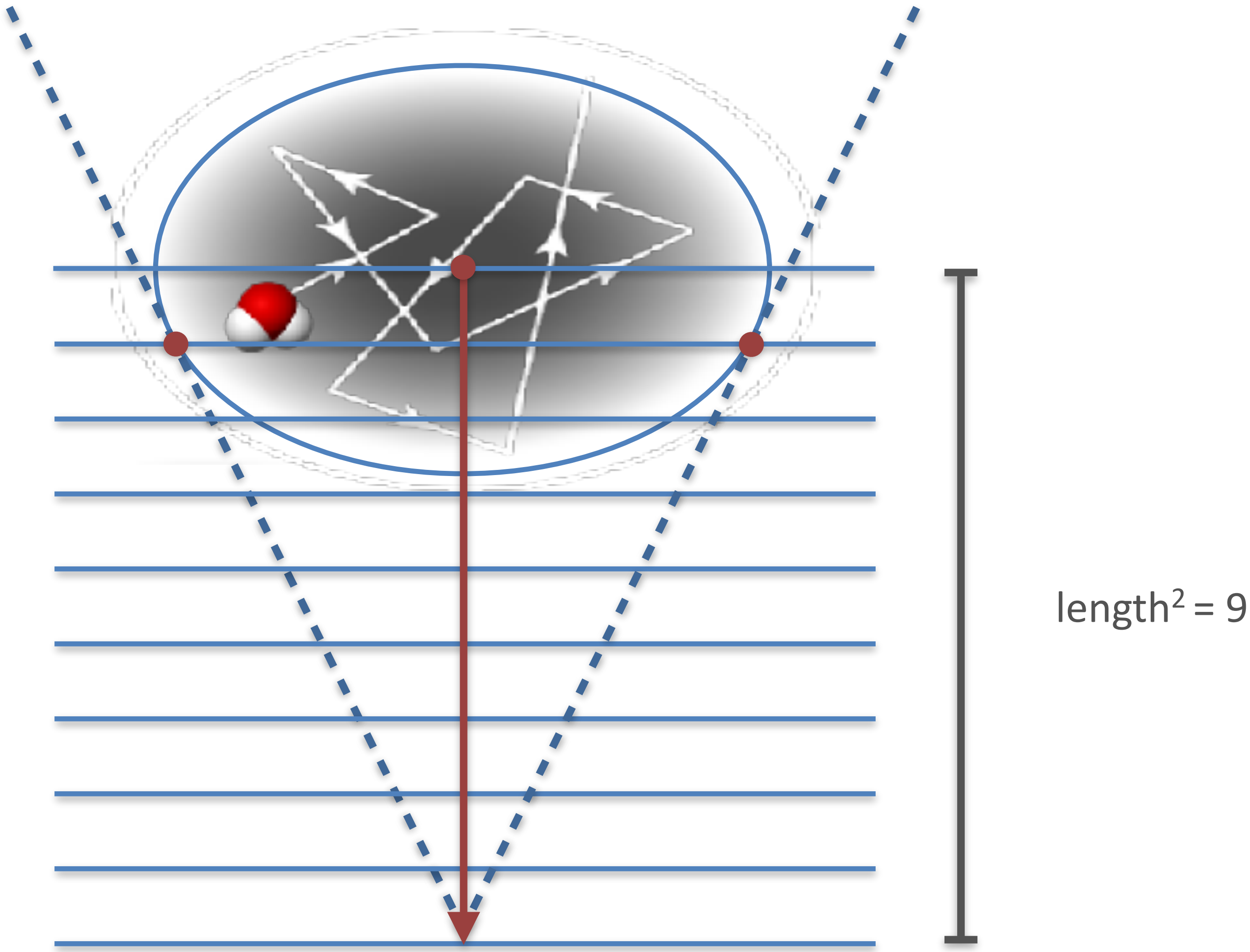
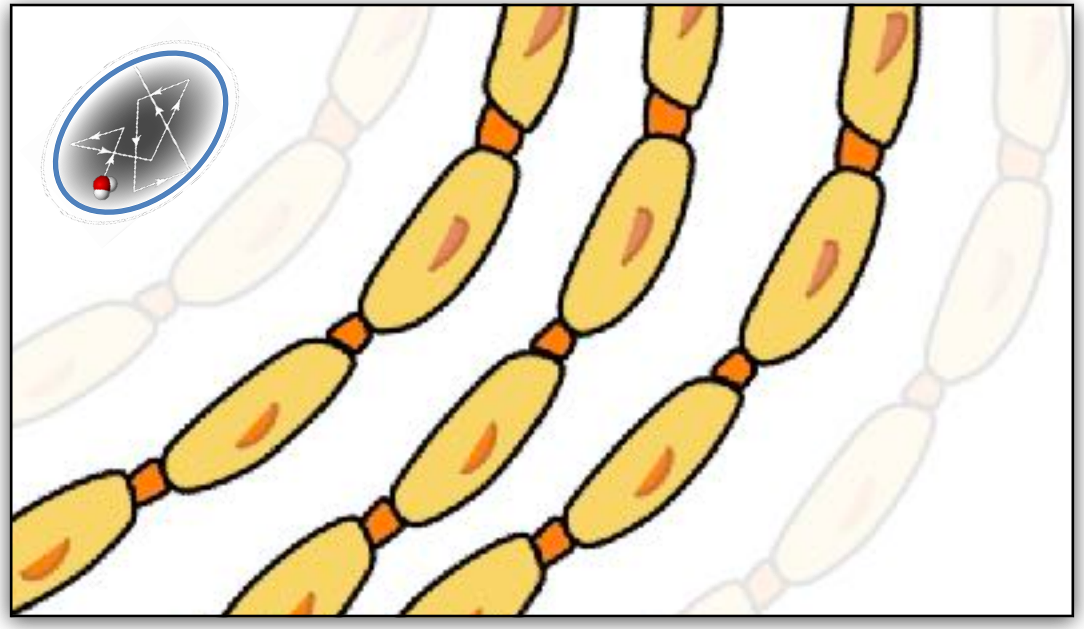


geodesic tractography



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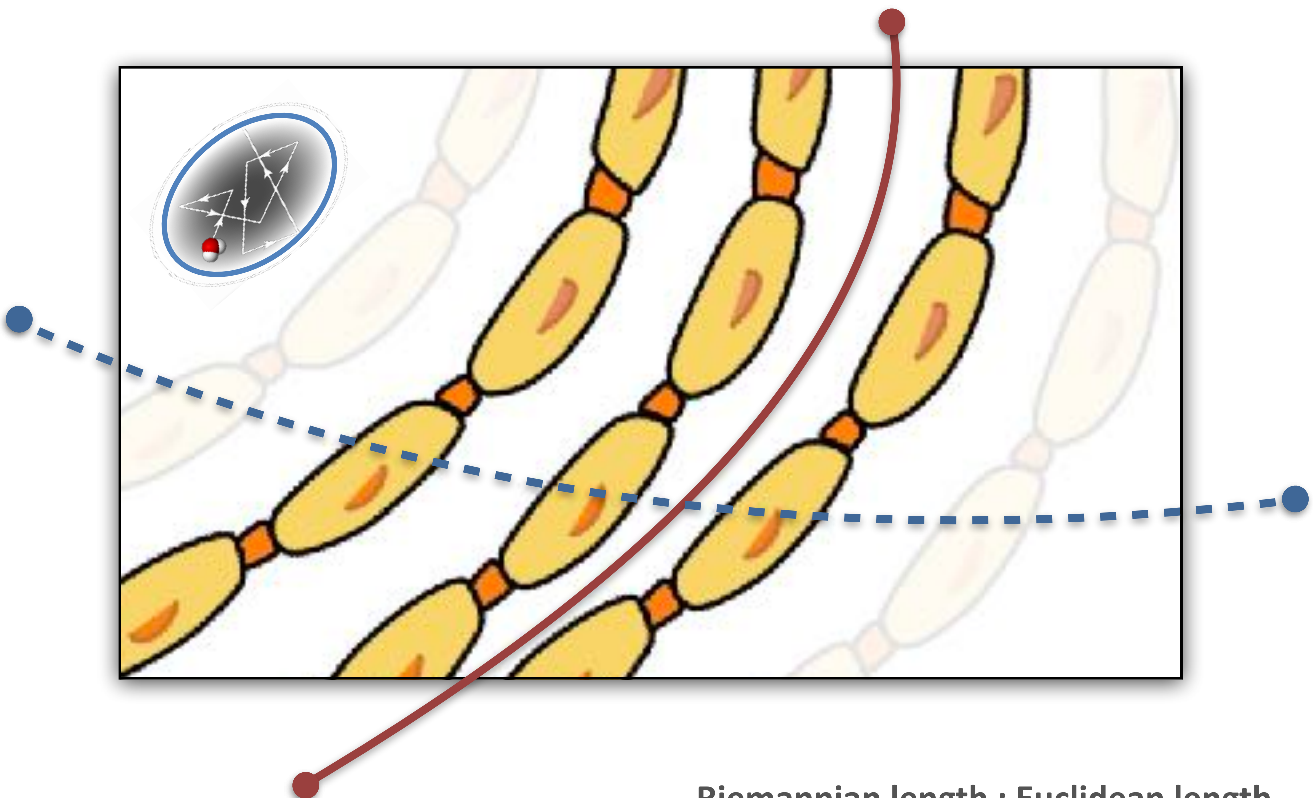

	Riemannian length : Euclidean length
'short' geodesic	5.0 : 6.0 < 1
'long' geodesic	7.5 : 6.0 > 1

Diffusion Tensor Imaging

versus

local gauge figure

Diffusion Tensor Imaging

physics pipeline

physics in a nutshell:

nuclear spin quantization → Zeeman splitting → Boltzmann statistics → magnetization → Bloch-Torrey equation → DTI

mathematics in a nutshell:

DTI → local gauge figure → geodesic tractography

Diffusion Tensor Imaging

physics pipeline

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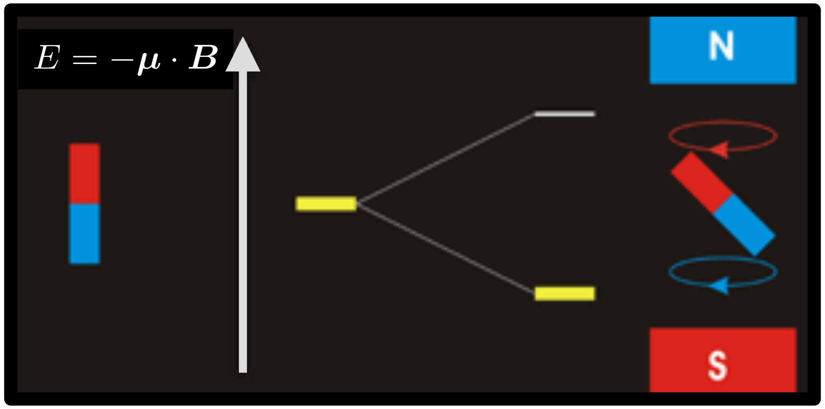
nuclear spin quantization → Zeeman splitting



$$E_{\downarrow} = +\frac{1}{2}\gamma\hbar B_z$$

$$E_{\uparrow} = -\frac{1}{2}\gamma\hbar B_z$$

$$\Delta E = \gamma\hbar B_z = \hbar\omega_{\mathrm{Larmor}}$$

Boltzmann statistics → magnetization

(typical clinical 3T MRI scanner)

$$\frac{N_\uparrow}{N_\downarrow} = \exp\left[\frac{\Delta E}{KT}\right] \approx 1 + \frac{\gamma \hbar B_z}{KT}$$

$$\frac{N_\uparrow - N_\downarrow}{N_\uparrow + N_\downarrow} \approx \frac{\gamma \hbar B_z}{2KT} \approx 10^{-5}$$

`low sensitivity modality’

$$M_z = \frac{N_\uparrow - N_\downarrow}{N_\uparrow + N_\downarrow} M_{\max} \approx (N_\uparrow + N_\downarrow) \frac{\gamma^2 \hbar^2 B_z}{4KT}$$

`big x small = measurable’

Bloch-Torrey equation → DTI

Bloch-Torrey / Stejskal-Tanner / Basser-Mattiello-Le Bihan:
Gaussian signal attenuation in q-space

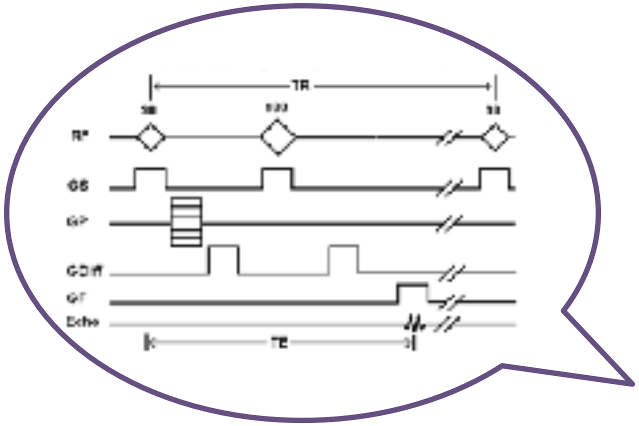
$$\frac{\partial M_\perp}{\partial t} = -i\gamma(M_\perp B_\parallel - M_\parallel B_\perp) - \frac{M_\perp}{T_2} + \nabla \cdot D \nabla M_\perp$$

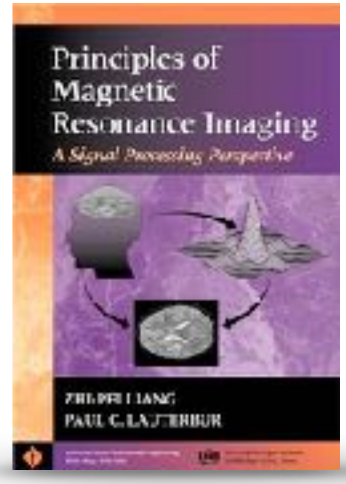
$$S(x, q, \tau) = S_0(x) \exp(-\tau q \cdot D(x) q)$$

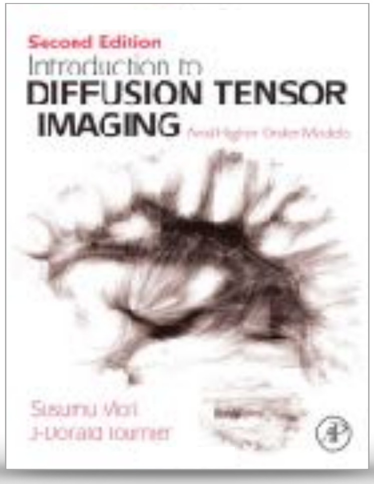
$$D(x) = -\frac{1}{\tau} \nabla_q^2 \ln \frac{S(x, q, \tau)}{S_0(x)}$$

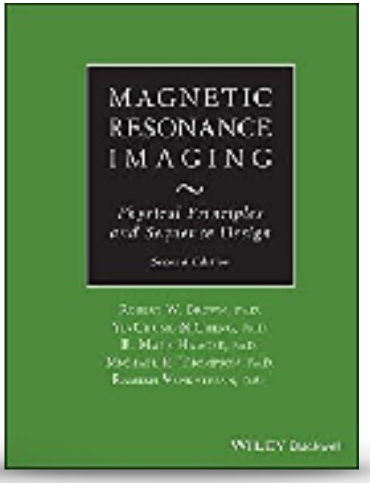
Bloch-Torrey equation → DTI

further reading









geodesic tractography

mathematics pipeline

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mathematics in a nutshell:

DTI → local gauge figure → geodesic tractography

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DTI → local gauge figure

DTI signal model: $$S(x,q,\tau) = S_0(x)e^{-\tau q^{\mathrm{T}} D(x) q}$$

Riemann metric: $$G(\xi,\xi)\,|_x = \xi^{\mathrm{T}} G(x)\xi$$

Lenglet et al. / O'Donnell et al.: $$G(x) \doteq D^{\mathrm{inv}}(x)$$

Fuster et al.: $$G(x) \doteq D^{\mathrm{adj}}(x)$$

local gauge figure → tractography

$$G(v,v) = \|v\|^2$$

Riemann metric: lengths & angles

$$\nabla_{\dot{x}}\dot{x} = 0$$

Levi-Civita connection: parallel transport

$$\ddot{x}^i + \sum_{jk} \Gamma^i_{jk}(x)\,\dot{x}^j\dot{x}^k = 0$$

Christoffel symbols: “pseudo-forces” (relative to local coordinate frames)

local gauge figure → tractography

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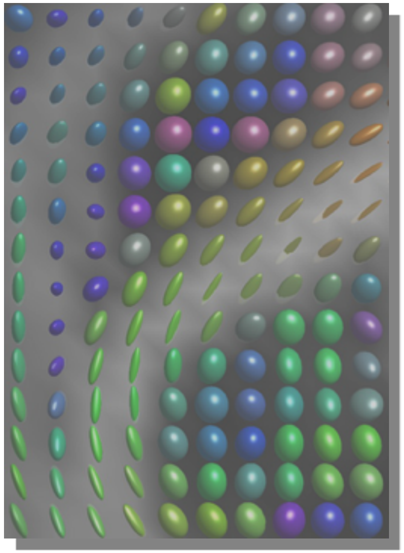

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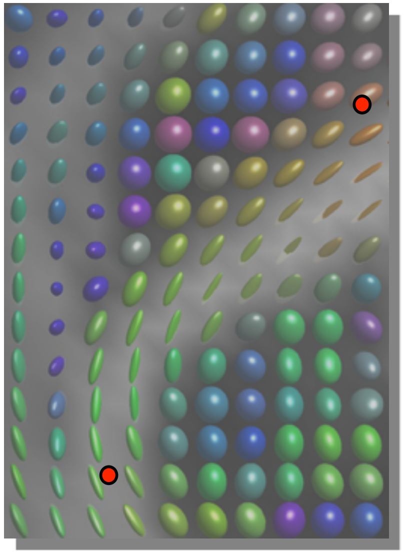

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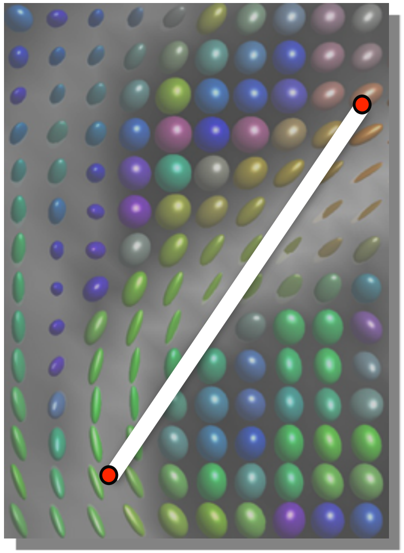

Euclidean geodesic

local gauge figure → tractography

$$G(v,v) = \|v\|^2$$

Riemann metric: lengths & angles

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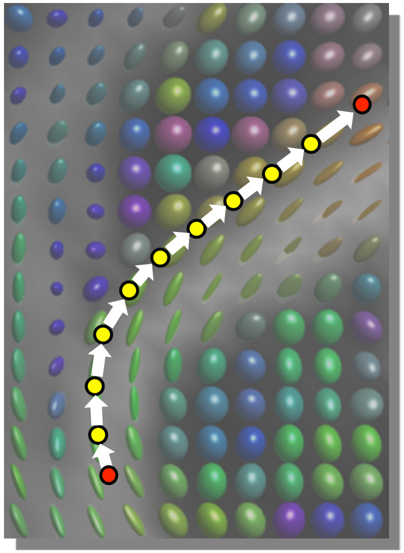


Riemannian geodesic

Riemann-DTI paradigm:

- neural fiber bundles correspond to relatively short geodesics in a Riemannian ‘brain space’
- the Riemannian structure can be inferred from DTI

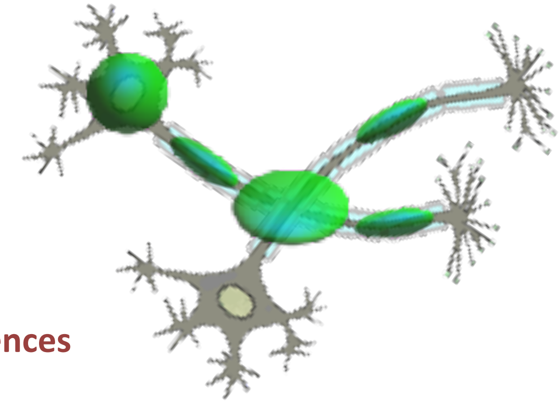
Riemann-DTI paradigm

pros & cons

geodesic completeness
=
redundant connections

pro: pixels → geodesic congruences

ellipsoidal gauge figure
=
poor angular resolution



con: destructive interference of orientation preferences

beyond the Riemann-DTI paradigm

a paradigm shift

specific model (DTI):

$$S(x,q,\tau) = S_0(x)\exp(-D(x,q,\tau))$$

with

$$D(x,q,\tau) = \tau q^{\mathrm{T}} D(x)\, q$$

6 d.o.f.'s per point sample
⬍
6 d.o.f.'s of local gauge figure

generic model (HARDI):

$$S(x,q,\tau) = \sum_{k=0}^{\infty} S^{i_1 \ldots i_k}(x,\tau)\phi_{i_1 \ldots i_k}(q)$$

or

∞ d.o.f.'s per point sample

$$D(x,q,\tau) = \sum_{k=0}^{\infty} D^{i_1 \ldots i_k}(x,\tau)\psi_{i_1 \ldots i_k}(q)$$

beyond the Riemann-DTI paradigm

a paradigm shift

$$\text{DTI} \subset \text{HARDI}$$

beyond the Riemann-DTI paradigm

a paradigm shift

DTI $\subset$ HARDI

⬍

Riemann geometry

beyond the Riemann-DTI paradigm

a paradigm shift

DTI $\subset$ HARDI

⬍ ⬍

Riemann geometry $\subset$ …

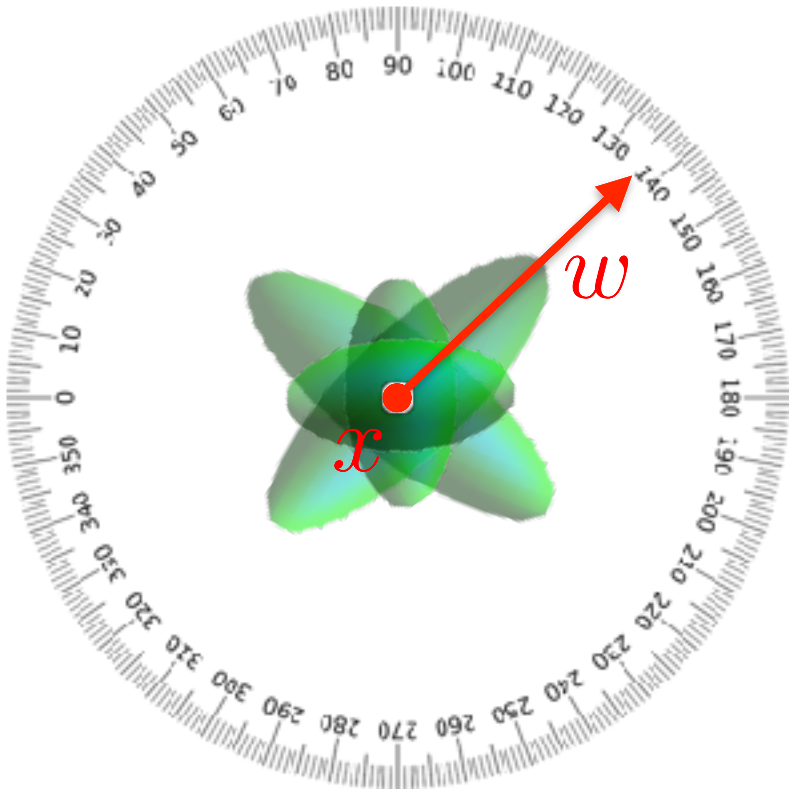
beyond the Riemann-DTI paradigm

a paradigm shift

DTI	$\subset$	HARDI
⬍		⬍
Riemann geometry	$\subset$	Finsler geometry

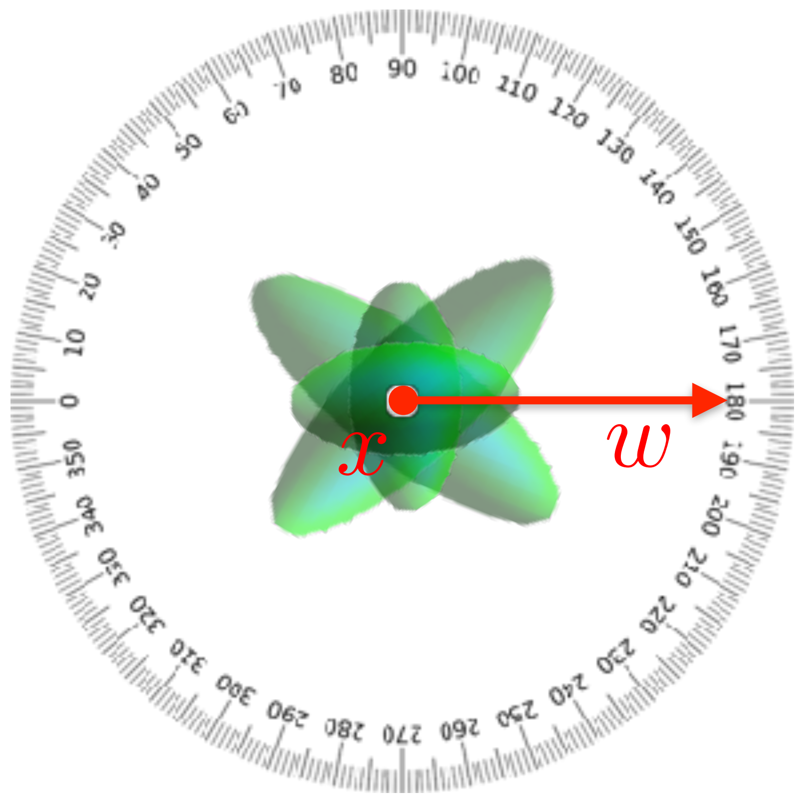
Finsler geometry

heuristics



Finsler geometry

heuristics



osculating indicatrices

base manifold: $x \in \mathbb{R}^3 \quad \rightarrow \quad (x, w) \in \mathbb{R}^3 \times \mathbb{S}^2$ **(sphere bundle)**

base manifold: $x \in \mathbb{R}^3 \quad \rightarrow \quad (x, \xi) \in \mathbb{R}^3 \times \mathrm{T}\mathbb{R}^3 \backslash \{0\}$ **(slit tangent bundle)**

Finsler geometry

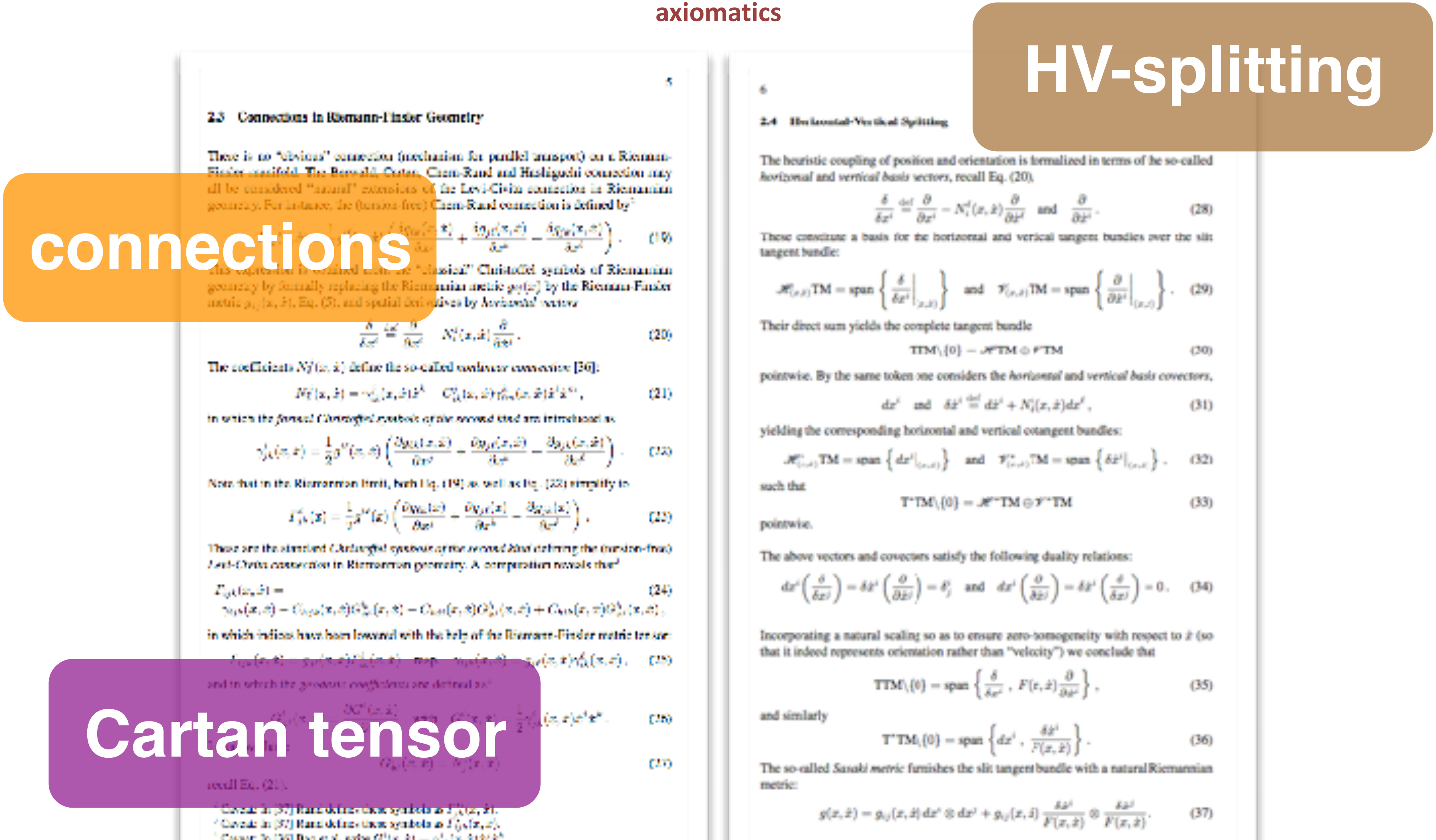
heuristics

family of osculating indicatrices
⬍
single (convex) indicatrix

gauge figure = unit sphere = indicatrix = Finsler metric ≠ inner product

Finsler geometry

axiomatics



in: Visualization and Processing of Tensors and Higher Order Descriptors for Multi-Valued Data, Springer 2014

Finsler geometry

axiomatics

Finsler metric: $$F^2(x,\xi) = g_{ij}(x,\xi)\xi^i\xi^j \qquad \Leftrightarrow \qquad g_{ij}(x,\xi) = \frac{1}{2}\partial_{\xi^i}\partial_{\xi^j}F^2(x,\xi)$$

Riemannian limit: $$F^2(x,\xi) = g_{ij}(x)\xi^i\xi^j \qquad \Leftrightarrow \qquad g_{ij}(x) = \frac{1}{2}\partial_{\xi^i}\partial_{\xi^j}F^2(x,\xi)$$

distance: $$d(x_1,x_2) = \inf\left\{\int_\gamma F(\gamma(t),\dot\gamma(t))\,dt \;\middle|\; \gamma\in C^1([t_1,t_2],\mathbb{R}^3)\ ,\ \gamma(t_1)=x_1\ ,\ \gamma(t_2)=x_2\right\}$$

Cartan tensor: $$C_{ijk}(x,\xi) = \frac{1}{2}\partial_{\xi^k}g_{ij}(x,\xi) = \frac{1}{4}\partial_{\xi^i\xi^j\xi^k}F^2(x,\xi)$$

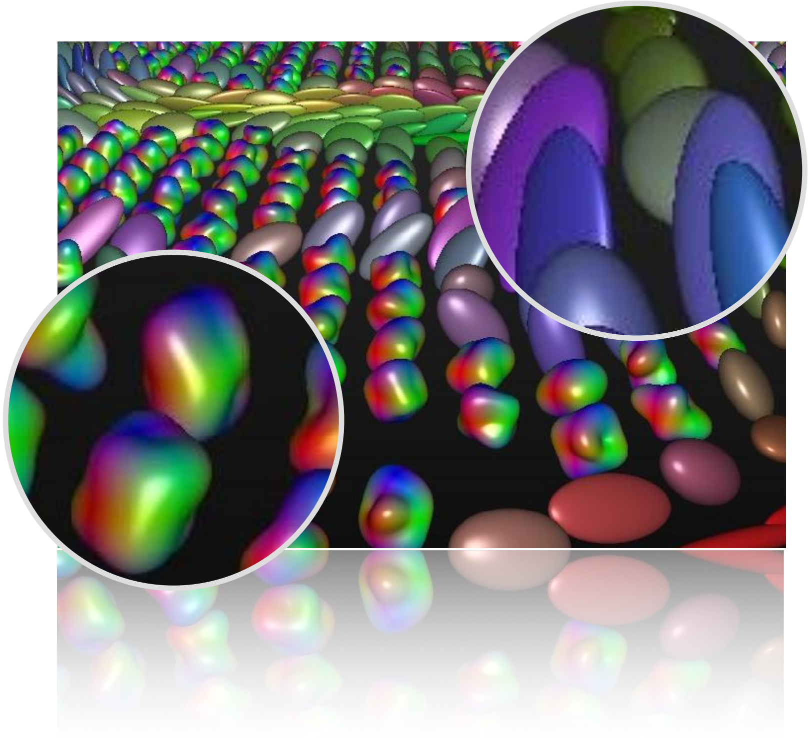
Riemannian limit: $$C_{ijk}(x,\xi) = 0$$ **(Deicke's Theorem)**

Finsler geometry

axiomatics

Deicke's Theorem:

Space is Riemannian iff the Cartan tensor vanishes.



Finsler-DTI paradigm

geodesic tractography

DTI ~ Riemannian geometry (inner product norm):

$$F^*(x,q) = \sqrt{q^{\mathrm{T}} \boldsymbol{D}(x) q}$$

$$F(x,y) = \sqrt{y^{\mathrm{T}} \boldsymbol{D}^{\mathrm{inv}}(x) y}$$

HARDI ~ Finslerian geometry (generalized norm):

$$F^*(x,\lambda q) = |\lambda| F^*(x,q)$$

$$F(x,\lambda y) = |\lambda| F(x,y)$$

Finsler-DTI paradigm

geodesic tractography

$$G_v(v,v) = \|v\|_F^2$$

Finsler metric: lengths

$$\nabla_{\dot{x}}\dot{x} = 0$$

Chern-Rund (or other) connection: parallel transport

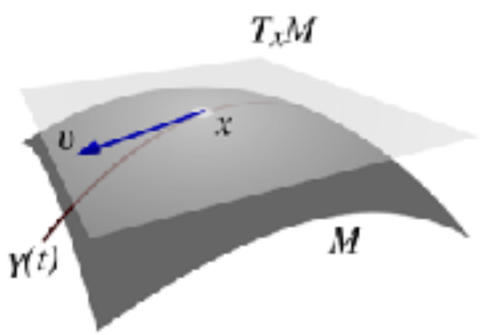
$$\ddot{x}^i + \sum_{jk} \Gamma^i_{jk}(x,\dot{x})\,\dot{x}^j\dot{x}^k = 0$$

formal Christoffel symbols: “pseudo-forces”

Finsler-DTI paradigm

operationalization*

- neural fiber bundles correspond to relatively short geodesics in the 3-dimensional 'horizontal part' of a 5-dimensional `brain space' furnished with a Finslerian structure
- this Finslerian structure can be inferred from diffusion MRI measurements
- the Finslerian dual metric can be interpreted as a '5-dimensional (3x3) DTI' tensor
- Finsler geometry encompasses Riemannian geometry as a special case
- the Finsler metric admits ∞ d.o.f.'s per spatial point as opposed to 6 d.o.f.'s for the Riemannian limit
- the Finsler-DTI paradigm admits versatile dimensionality reduction in trade-off with acquisition time



* Tom Dela Haije, PhD thesis, May 16 2017, Eindhoven

Finsler-DTI paradigm

summary

Finsler function:

$$F^*(x, \lambda q) = |\lambda| F^*(x, q)$$

$$F(x, \lambda y) = |\lambda| F(x, y)$$

Finsler-DTI paradigm

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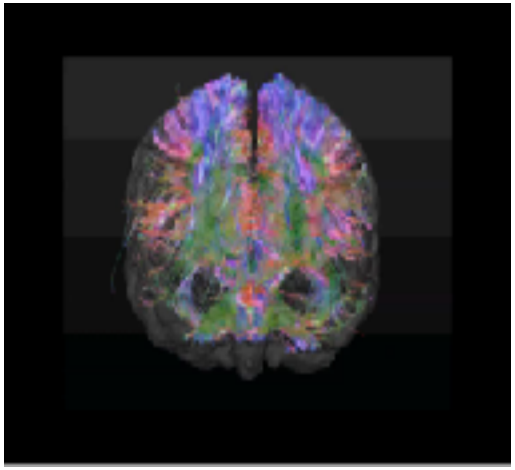
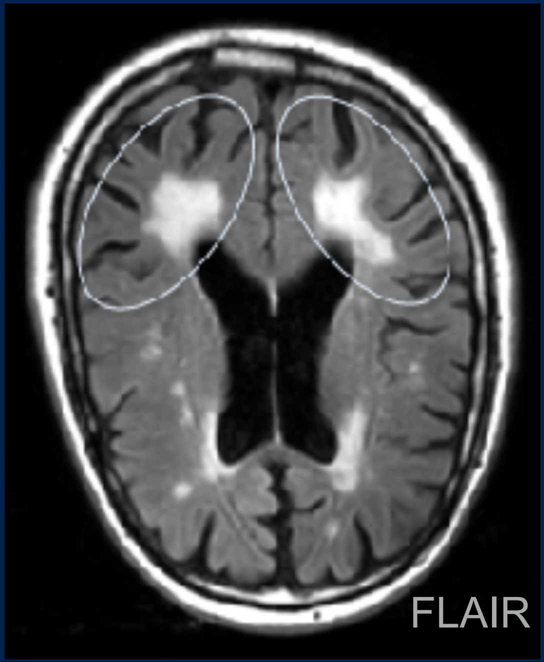
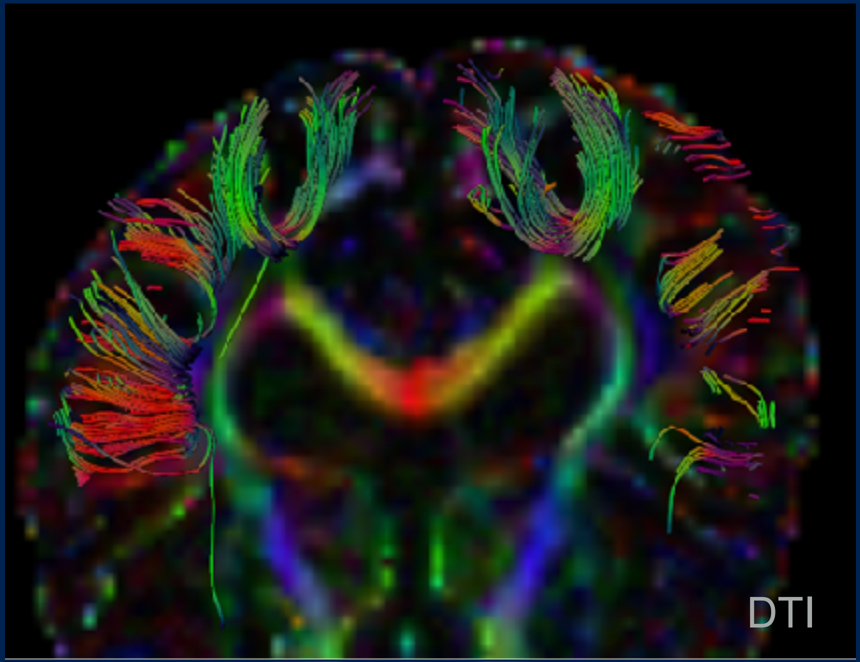
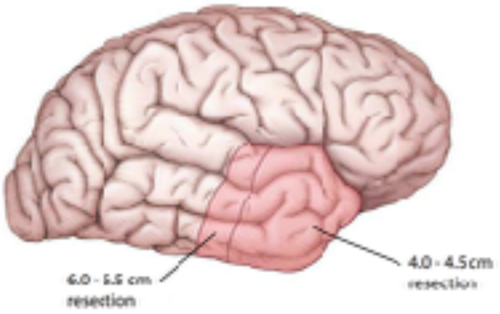


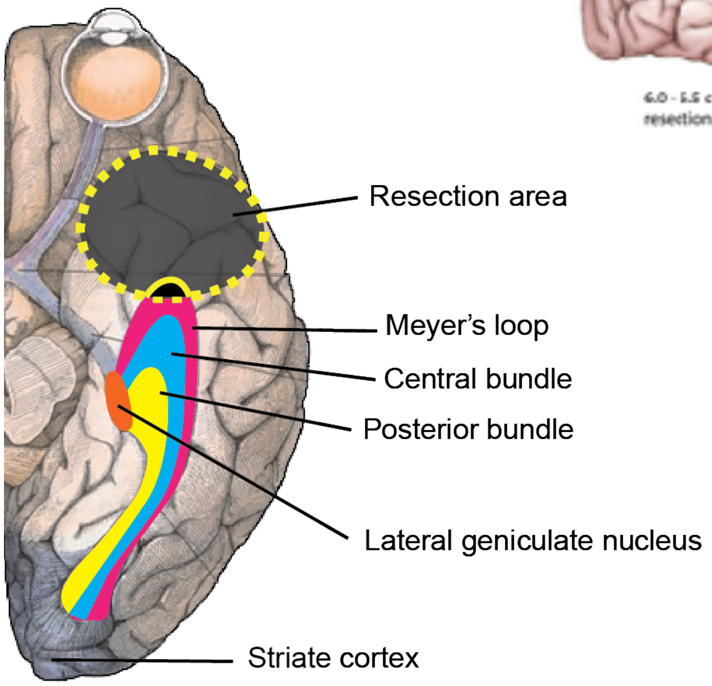
applications & outlook

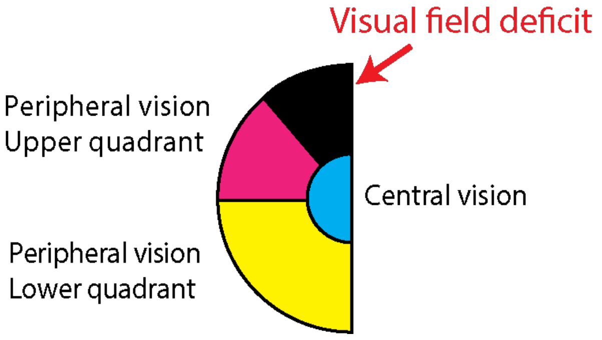




applications & outlook

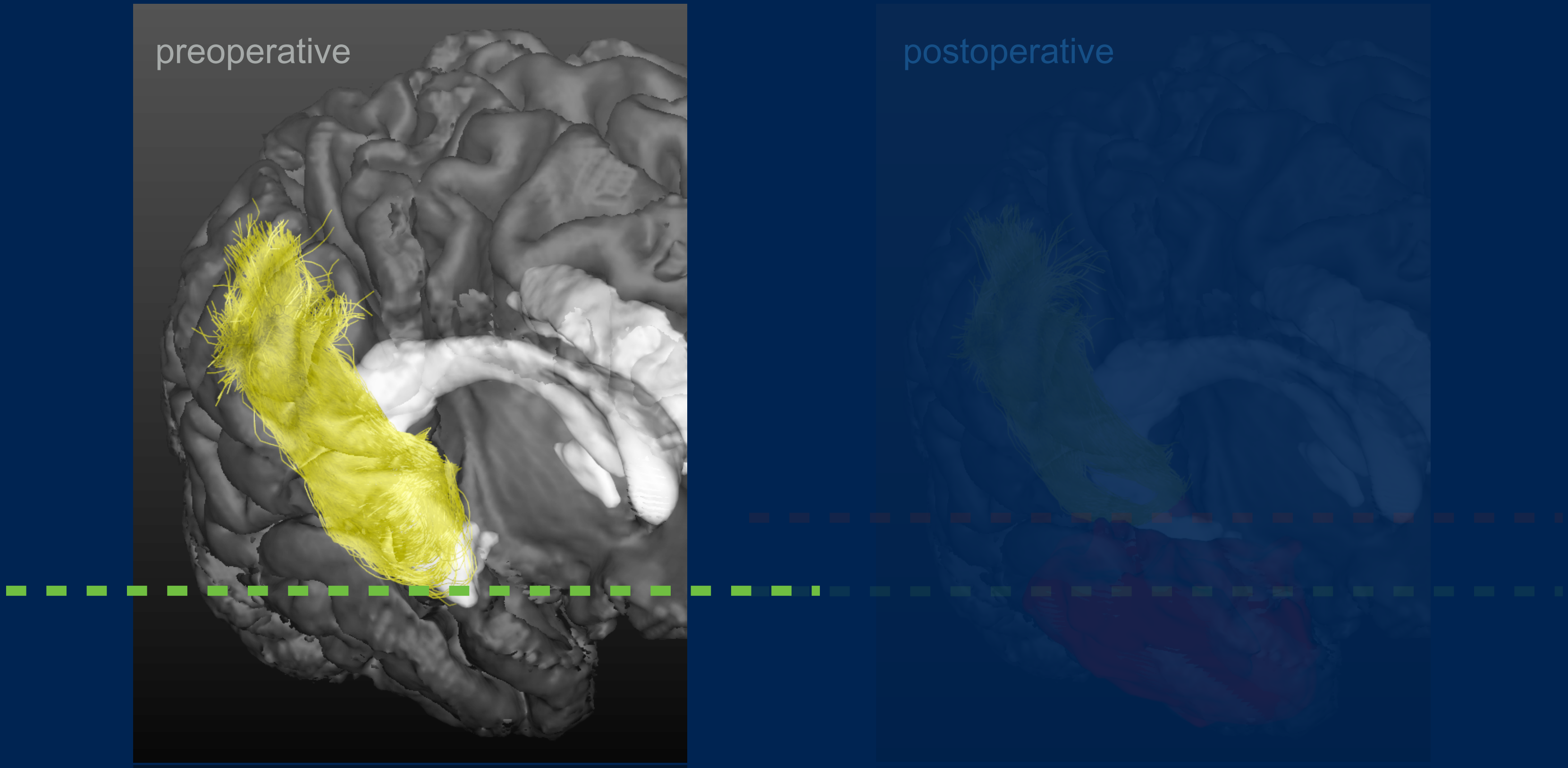




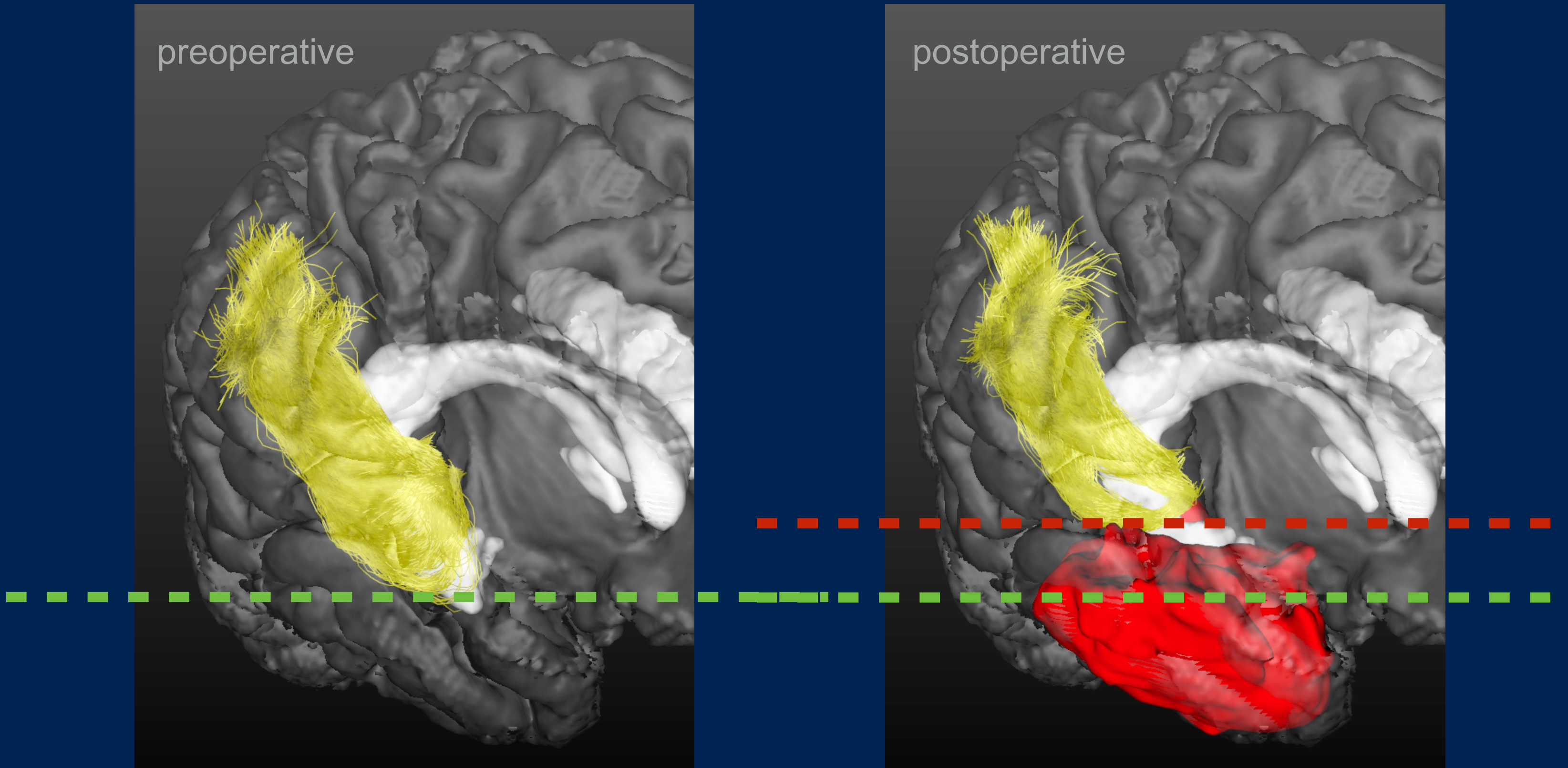


Stephan Meesters et al., electronic poster 3476, ISMRM 2017, Hawaii

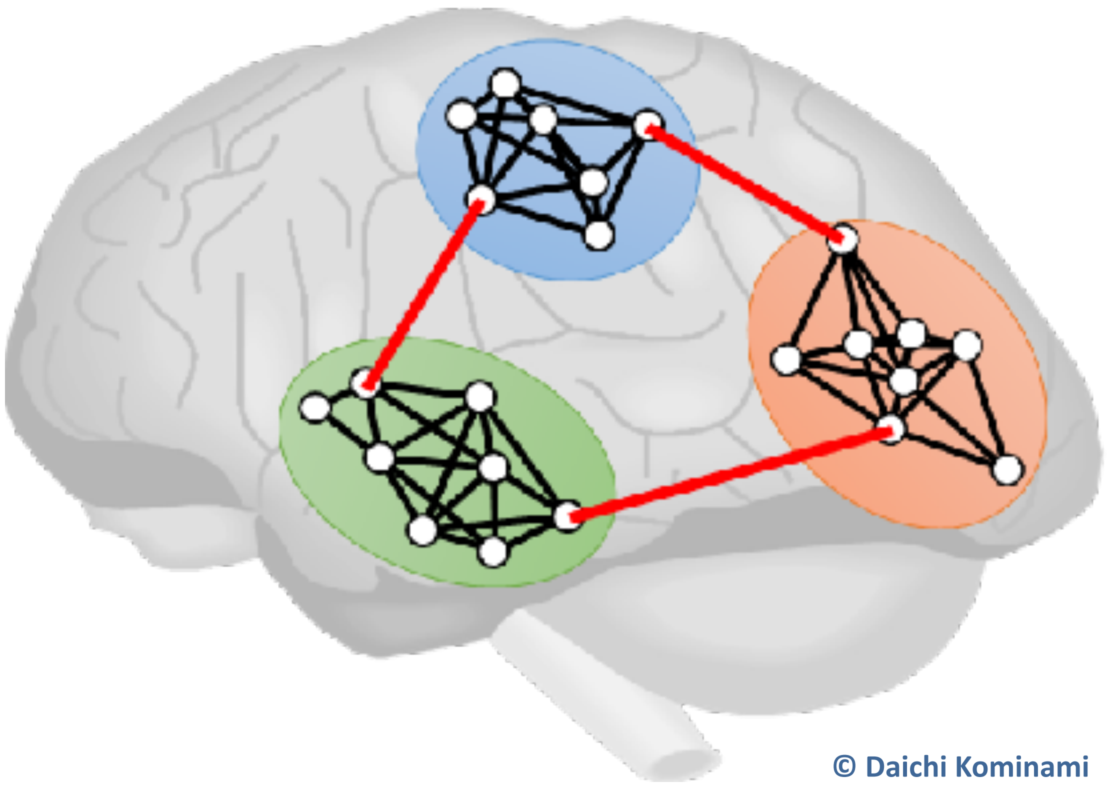
applications & outlook



applications & outlook

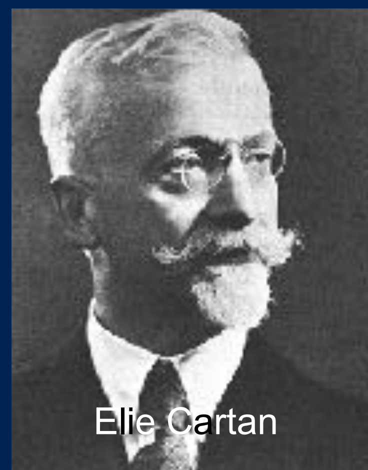


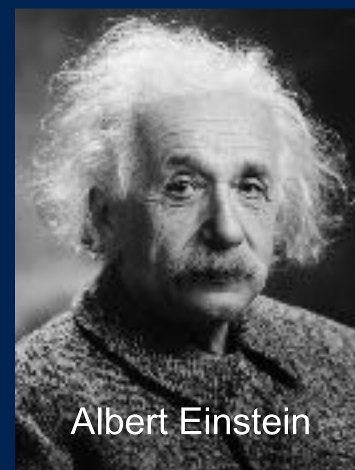
applications & outlook

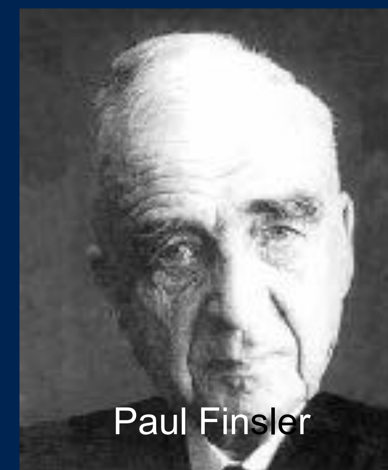


conclusion

- Finsler geometry is a generic and potentially powerful framework for diffusion MRI beyond classical DTI
- this framework allows us to exploit a rich body of knowledge gained over more than a century by great scientists









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